

BEAMING IN GAMMA-RAY BURSTS: EVIDENCE FOR A STANDARD ENERGY RESERVOIR

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ABSTRACT

We present a comprehensive sample of all gamma-ray burst (GRB) afterglows with known distances, and we derive their conical opening angles based on observed broadband breaks in their light curves. Within the framework of this conical jet model, we correct for the geometry and we find that the gamma-ray energy release is narrowly clustered around 5×10^{50} ergs. We draw three conclusions. First, the central engines of GRBs release energies that are comparable to ordinary supernovae. Second, the broad distribution in fluence and luminosity for GRBs is largely the result of a wide variation of opening angles. Third, only a small fraction of GRBs are visible to a given observer, and the true GRB rate is several hundred times larger than the observed rate.

Subject headings: gamma rays: bursts — ISM: jets and outflows — shock waves

1. INTRODUCTION

One of the keys to understanding the progenitors of gamma-ray bursts (GRBs) and the physics of their central engines lies in determining the energetics of the explosion. The isotropic equivalent gamma-ray energy $E_{\text{iso}}(\gamma) = 4\pi F_{\gamma} d_L^2 (1+z)^{-1}$ has been used as a surrogate for the energy released by the central engine, where F_{γ} is the fluence of the burst, z is the redshift, and d_L is the luminosity distance.

Although jets⁶ in GRBs were first suggested for GRB 970508 (Waxman, Kulkarni, & Frail 1998), they were widely evoked for GRB 990123 (e.g., Fruchter et al. 1999) to explain its spectacular energy release. Subsequent multiwavelength observations of GRBs have been interpreted as evidence for explosions with jetlike geometry (Stanek et al. 1999; Harrison et al. 1999). The detection of polarization (e.g., Covino et al. 1999; Wijers et al. 1999) gave further credence to the jet hypothesis: the nonspherical geometry leads to polarized signal, from which the geometry of the jet can be inferred (Ghisellini & Lazzati 1999; Sari 1999).

The signature of conical geometry manifests itself as a broadband “break” in the power-law decay of the afterglow emission, which declines more rapidly relative to that of a spherical case (e.g., Rhoads 1997; Sari, Piran, & Halpern 1999). This break happens for two reasons. The first is an edge effect that occurs at a time t_j when the bulk Lorentz factor of the blast wave (Γ) has slowed down to $\Gamma < \theta_j^{-1}$ (where θ_j is the opening angle of the jet). The second effect is the lateral spreading of the jet. The ejecta, now encountering more surrounding matter, decelerate faster than in the spherical case.

Here we carry out an analysis of jet opening angles, determined in a simple but consistent framework, which we use to

estimate the true energy release as well as obtain the beaming-corrected GRB rate. The uncertainties and possible limitations of our method are discussed in § 6.

2. SAMPLE OF CONICAL AFTERGLOWS

In Table 1, we present a complete sample of 17 GRBs with known redshifts as of 2001 January. The jet break times t_j are taken directly from the literature and have uncertainties of order 10%–30%. The best events are those for which it is possible to globally model the broadband data within the physical framework of the relativistic jet model (“B” in Table 1). For example, GRB 990510 exhibited a jetlike signature at optical and radio (Stanek et al. 1999; Harrison et al. 1999) wavelengths, which was found to be consistent with the X-ray light curve (Pian et al. 2001). In most cases, such multifrequency data sets are not available, so there is a second class of events with breaks determined primarily from radio (R), optical (O), or X-ray (X) data. Such breaks may not be uniquely attributed to jets (see § 6). We include here a number of events for which no break was observed, yielding only lower limits of t_j . For some GRBs, the steep decline of the light curve, indicating a jet geometry, is already fully manifest at the time of the first measurement. In these cases (D), we have only an upper limit on t_j . The final group of GRBs are those for which t_j cannot be determined (N) owing to complications in the light curve such as the presence of a supernova signature (i.e., GRB 970228) or the lack of sufficient data.

3. DETERMINATION OF JET OPENING ANGLES

To convert the measured jet break times t_j to opening angles of the conical blast wave, we used the formulation of Sari et al. (1999):

$$\theta_j = 0.057 \left(\frac{t_j}{1 \text{ day}} \right)^{3/8} \left(\frac{1+z}{2} \right)^{-3/8} \left[\frac{E_{\text{iso}}(\gamma)}{10^{53} \text{ ergs}} \right]^{-1/8} \times \left(\frac{\eta_{\gamma}}{0.2} \right)^{1/8} \left(\frac{n}{0.1 \text{ cm}^{-3}} \right)^{1/8}, \quad (1)$$

where η_{γ} is the efficiency of the fireball in converting the energy in the ejecta into gamma rays and n is the mean circumburst

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⁶ Henceforth, following standard usage, we will interchangeably use the term “jet” for conical blast waves.

TABLE 1
JET BREAK TIMES AND ENERGETICS

GRB	F_γ	z	d_L	$E_{\text{iso}}(\gamma)$	t_j	θ_j	E_γ	Reference	Method
970228	11.0	0.695	1.4	22.4	N
970508	3.17	0.835	1.8	5.46	25	0.293	0.234	1	R
970828	96.0	0.958	2.1	220	2.2	0.072	0.575	2	X
971214	9.44	3.418	9.9	211	>2.5	>0.056	>0.333	3	O
980613	1.71	1.096	2.5	5.67	>3.1	>0.127	>0.045	4	O
980703	22.6	0.966	2.1	60.1	7.5	0.135	0.544	5	B
990123	268	1.600	3.9	1440	2.04	0.050	1.80	6	O
990506	194	1.30	3.0	854	N
990510	22.6	1.619	4.0	176	1.20	0.053	0.248	7	B
990705	93	0.84	1.8	270	~1	0.054	0.389	8	O
990712	6.5	0.433	0.8	5.27	>47.7	>0.411	>0.445	9	O
991208	100	0.706	1.4	147	<2.1	<0.079	<0.455	10	D
991216	194	1.02	2.3	535	1.2	0.051	0.695	11	O
000131	41.8	4.500	13.7	1160	<3.5	<0.047	<1.30	12	D
000301C	4.1	2.034	5.3	46.4	5.5	0.105	0.256	5	B
000418	20.0	1.119	2.5	82.0	25	0.198	1.60	13	B
000926	6.2	2.037	5.3	297	1.45	0.051	0.379	14	O

NOTES.—Fluence ($F_\gamma/10^{-6}$ ergs cm^{-2}), redshift (z), luminosity distance ($d_L/10^{28}$ cm), and isotropic gamma-ray energies [$E_{\text{iso}}(\gamma)/10^{51}$ ergs] are taken from Bloom et al 2001. Jet break times (t_j /days) were determined from the listed references, using the methods as discussed in the text. The geometry-corrected gamma-ray energy E_γ is given in units of 10^{51} ergs.

REFERENCES.—(1) Frail et al. 2000; (2) Djorgovski et al. 2001; (3) Kulkarni et al. 1998; (4) Halpern & Fesen 1998; (5) E. Berger et al. 2001, in preparation; (6) Kulkarni et al. 1999; (7) Harrison et al. 1999; (8) Masetti et al. 2000; (9) Fruchter et al. 2000; (10) Jensen et al. 1999; (11) Halpern et al. 2000; (12) Andersen et al. 2000; (13) Berger et al. 2001; (14) Price et al. 2001.

density. A number of recent papers (Beloborodov 2000; Guetta, Spada, & Waxman 2001; Kobayashi & Sari 2001) have argued that internal shocks under certain conditions are very efficient at producing gamma rays ($\eta_\gamma \gtrsim 0.2$). Likewise, broadband modeling of GRB afterglows (Frail, Waxman, & Kulkarni 2000; Panaitescu & Kumar 2001) give estimates of gas densities consistent with galactic disks. These agreements justify our particular normalization in equation (1).

Using equation (1), we obtain a range in θ_j corresponding to the wide range in t_j -values in Table 1 (from $\lesssim 1$ day to 30 days). The derived jet angles vary from 1° to more than 25° , with a strong concentration near 4° (Fig. 1). It is reasonable to ask whether the observed distribution in Figure 1 suffers from selection effects. To begin, we note that out of the 21 known optical afterglows, the light curves of only two GRBs—GRB 980326 (Groot et al. 1998) and GRB 980519 (Jaunsen et al. 2001)—show rapid decline implying $t_j \lesssim 1$ day. Likewise, out of a sample of 10 bright X-ray afterglows observed with the *BeppoSAX* satellite, there is no evidence for a significant break within 8–48 hr after a burst (Stratta et al. 2000), suggesting that $t_j \gtrsim 1$ day for these events. If we increase the sample to include the 28 GRBs detected by *BeppoSAX* for which follow-up searches (typically 8–12 hr after the burst) were made for an X-ray afterglow, we find only one unambiguous case where no afterglow was detected, i.e., GRB 990217 (Costa 2000). There are a further six cases where a hitherto uncataloged X-ray source was detected in the GRB error circle. In every case the X-ray source is a plausible afterglow but lacking multiwavelength confirmation, the afterglow identification remains uncertain, e.g., GRB 970111 (Costa 2000). From these statistics, we conclude that steep decays, $t_j \lesssim 1$ day, and therefore very narrow opening angles, $\theta_j < 3^\circ$, are required for less than 10% of the *BeppoSAX* GRB sample.

GRBs with large opening angles do not suffer from severe beaming, but it is not easy to measure t_j for such bursts. For large t_j , the afterglow emission is weak and (at optical wavelengths) the host galaxy starts dominating (e.g., Halpern et al. 2000). Thus, optical and X-ray observations are unlikely to

yield t_j . Fortunately, radio observations can and do play a crucial role, due to the long lifetime of the afterglow in this regime. This was the case for four out of five wide-angle jets identified in Table 1. One recent example is GRB 000418, where a jet break inferred from radio measurements (Berger et al. 2001) was confirmed by late-time *Hubble Space Telescope* observations (Fruchter & Metzger 2001).

4. GEOMETRIC CORRECTIONS AND BURST ENERGETICS

A conical GRB will not light up the full celestial sphere but rather a fraction, the so-called beaming fraction (Rhoads 1999; Sari et al. 1999) $f_b = (1 - \cos \theta_j) \approx \theta_j^2/2$. Thus, the true gamma-ray energy released, E_γ , is smaller than $E_{\text{iso}}(\gamma)$ by the same factor, i.e., $E_\gamma = f_b E_{\text{iso}}(\gamma)$ (Fig. 2). Applying the Bayesian inference formalism (Reichart et al. 2001), we find that $\langle E_{\text{iso}}(\gamma) \rangle$ is 110×10^{51} ergs with a 1σ spreading of a multiplicative factor of 6.2, while E_γ is clustered around 5×10^{50} ergs, with a 1σ multiplicative factor of only 2. Thus, it appears that the central engines of GRBs produce approximately a similar amount of energy and a significant part, about 10^{51} ergs, escapes as gamma rays (Fig. 2). However, for reasons not presently understood, there exists a wide range of jet opening angles (Fig. 1), and as noted previously (Sari et al. 1999), this results in the apparent wide distribution of fluence.

The narrowness of the E_γ distribution is surprising and has several immediate implications. While it is not unreasonable to expect that the central engines produce a similar amount of energy, E_0 , in each explosion, there is little reason to expect that they will produce similar gamma-ray outputs. Since the true total energy $E_0 \equiv E_\gamma/\eta_\gamma \propto n^{1/4} \eta_\gamma^{-3/4}$ (this follows from eq. [1]), the narrowness in the distribution of E_γ places restrictions on the dispersion of n and η_γ . Finally, the narrowness of the E_γ distribution requires that the brightness of the gamma-ray beam be roughly uniform from the center to the edge. This is contrary to models (Kumar & Piran 2000) in which large intensity variations within the conical blast wave are invoked in order to explain the wide dispersion of peak luminosities.

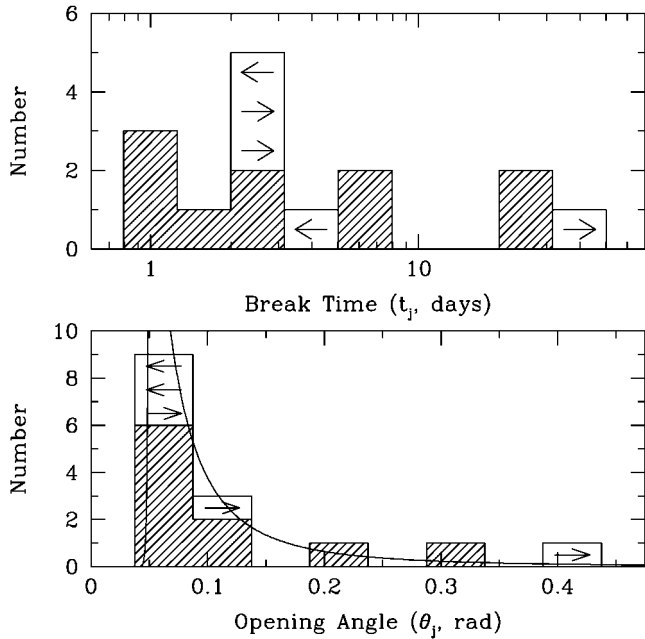


FIG. 1.—Observed distribution of jet break times (*top panel*) and jet opening angles (*bottom panel*). For the model fit (*solid line*), we assume that the observed differential distribution of beaming factors can be represented by two power laws: $p_{\text{obs}}(f_b) = (f_b/f_0)^{\alpha+1}$ for $f_b < f_0$ and $p_{\text{obs}}(f_b) = (f_b/f_0)^{\beta+1}$ for $f_b > f_0$. Since for every observed burst there are f_b^{-1} that are not observed, the true distribution is $p_{\text{true}}(f_b) = f_b^{-1} p_{\text{obs}}(f_b)$. Fitting to the data, we find the following: α is poorly constrained; $\beta = -2.77_{-0.30}^{+0.24}$, and $\log f_0 = -2.91_{-0.06}^{+0.07}$. Thus, the true differential probability distribution (under the small angle approximation, $f_b \propto \theta_j^2$) is given by $p_{\text{true}}(\theta_j) \propto \theta_j^{-4.54}$ with the observed distribution being $p_{\text{obs}} \propto \theta_j^{-2.54}$. The distribution $p_{\text{true}}(f_b)$ allows us to estimate the true correction factor, $\langle f_b^{-1} \rangle$ that has to be applied to the observed GRB rate in order to obtain the true GRB rate. We find $\langle f_b^{-1} \rangle = f_0^{-1}[(\beta - 1)/\beta] \sim 520 \pm 85$.

We find that most of the dispersion in the luminosity is due to the diversity in opening angles.

The mean value of E_γ is 5×10^{50} ergs (Fig. 2). If we use the estimate of Guetta et al. (2001), then $\eta_\gamma \sim 0.2$ and we derive $E_0 \sim 3 \times 10^{51}$ ergs. This energy is only slightly larger than the typical 10^{51} ergs of electromagnetic and kinetic energy yield of ordinary supernovae. The mystery about GRBs is no longer in understanding their supposedly extraordinary energy budget but in explaining why the ejecta of GRBs have such a high Lorentz factor.

Of course, E_0 is sensitive, in addition to the adopted value of η_γ , to the overall scaling, i.e., the numerical coefficient of equation (1). Fortunately, GRB 970508 allows us to directly determine the energy scale. The radio afterglow of this GRB lasted long enough (400 days) that the blast wave was non-relativistic, thereby allowing determination of the total energy (Frail et al. 2000) independent of relativistic beaming. Table 1 shows that this burst has one of the lowest energies, although it is only 1σ away from the mean (if the energy distribution is assumed to be lognormal). The agreement between these two entirely different approaches is remarkably good and gives some support to our choice of the numerical coefficient and normalization of equation (1).

5. BEAMING FRACTION AND THE GRB RATE

Since conical fireballs are visible to only a fraction, f_b , of observers, the true GRB rate $R_t = \langle f_b^{-1} \rangle R_{\text{obs}}$, where R_{obs} is the observed GRB rate and $\langle f_b^{-1} \rangle$ is the harmonic mean of the beaming fractions. We find $\langle f_b^{-1} \rangle \sim 500$ (see Fig. 1). The formal

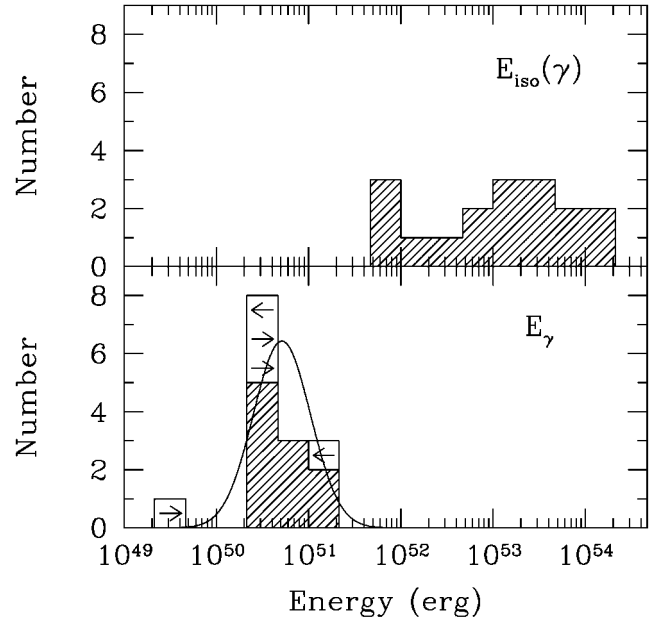


FIG. 2.—Distribution of the apparent isotropic GRB energy of GRBs with known redshifts (*top panel*) vs. the geometry-corrected energy for those GRBs whose afterglows exhibit the signature of a nonisotropic outflow (*bottom panel*). Arrows are plotted for five GRBs to indicate upper or lower limits to the geometry-corrected energy.

uncertainty in this estimate is only 16%, but systematic uncertainties related to our choice of the numerical coefficient and normalization of equation (1) make this estimate accurate to no more than a factor of 2.

Following Schmidt (2001), we assume $R_{\text{obs}}(z = 0) = 0.5 \text{ Gpc}^{-3} \text{ yr}^{-1}$. The true rate is $R_t(z = 0) \sim 250 \text{ Gpc}^{-3} \text{ yr}^{-1}$, which should be compared with the estimated rate (Phinney 1991) of neutron star coalescence, $R_c(z = 0) \sim 80 \text{ Gpc}^{-3} \text{ yr}^{-1}$ and the estimated rate (Phinney 1991) of Type Ibc supernovae, $R_{\text{Ibc}} \sim 6 \times 10^4 \text{ Gpc}^{-3} \text{ yr}^{-1}$. Clearly, the collapsar scenario is easily capable of supplying a sufficient number of progenitors. Within the uncertainties of the estimates, the coalescence scenario is (just barely) capable of providing sufficient progenitors.

6. DISCUSSIONS AND CONCLUSIONS

The framework we have used in determining the geometry and energetics of GRBs is a simple one. Given the limitations of the existing data, it is the only method that is available to analyze the full sample of GRBs with known redshifts. This method has led us to conclude that there exists a distribution of t_b - and θ_j -values and that this inevitably leads to a reduction in the gamma-ray energy from its isotropic value. The correctness of the conclusions in this Letter will be tested by future well-studied GRB events and more detailed panchromatic modeling of GRB afterglows. Although the availability of such high-quality radio, optical, and X-ray data sets needed to carry out this type of analysis are rare and growing slowly, preliminary efforts (Berger et al. 2000; Panaitescu & Kumar 2001; Harrison et al. 2001) are building the case for a collimated outflows in low-density ($n \sim 1 \text{ cm}^{-3}$) media with total energies of order $\sim 10^{51}$ ergs.

Freedman & Waxman (2001) and Kumar (2000) have suggested an elegant way to estimate the fireball energy based on X-ray afterglow observations. While this complementary method is less sensitive to our assumptions of constant density

n and efficiency η_γ and of jet uniformity, it is instead sensitive to inverse Compton effects and radiative losses during the early afterglow. Remarkably, applying our determinations of f_b to the six GRBs from (Freedman & Waxman 2001) in common with Table 1, we obtain $E_a = (2.7 \pm 1.4) \times 10^{50}$ ergs. Within the limitations of this small sample, the results are in agreement with our findings (Fig. 2).

We end with a number of caveats. Our derivation of the opening angle is based on equation (1), which implicitly assumes that GRBs explode in a constant density medium and that any sharp break in the afterglow is attributed to a combination of the observer viewing beyond the edge of the conical jet and sideways expansion. The origin of the observed breaks is currently a matter of considerable theoretical debate. The uncertainty is driven by the as yet unclear hydrodynamics of sideways expansion (e.g., Moderski, Sikora, & Bulik 2000). Other mechanisms have been proposed to produce steep declines in the afterglow light curves: (1) a sudden drop in the external density (Kumar & Panaitescu 2000), (2) a transition from a relativistic to nonrelativistic regime (Wang, Dai, & Lu 2000) due to expansion in a dense circumburst medium, and (3) a break in the power-law distribution of radiating electrons (Li & Chevalier 2001). However, we note that at this stage the simple jet model, which we have adopted, provides a consistent and adequate description of the observations. Within the framework of this simple jet model, we have deduced the distribution of the opening angles of GRB jets and empirically uncovered

what may be an important clue, namely, the total energy release and its approximate constancy.

Note added in manuscript.—There has been one well-studied GRB since our analysis was carried out. GRB 010222 was the second brightest burst (as measured by fluence) in the 5 yr mission of *BeppoSAX*. It was rapidly localized ($z = 1.477$), and an achromatic break in its light curve was identified at $t_j \approx 0.72$ days. Using equation (1), Stanek et al. (2001) derive $E_\gamma = 4 \times 10^{50}$ ergs—a value in good agreement with $\langle E_\gamma \rangle$.

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REFERENCES

- Andersen, M. I., et al. 2000, *A&A*, 364, L54
 Beloborodov, A. M. 2000, *ApJ*, 539, L25
 Berger, E., et al. 2000, *ApJ*, 545, 56
 ———. 2001, *ApJ*, 556, 556
 Bloom, J. S., Frail, D. A., & Sari, R. 2001, *AJ*, 121, 2879
 Costa, E. 2000, in *Fifth Huntsville Symp. on Gamma-Ray Bursts*, ed. R. M. Kippen, R. S. Mallozzi, & G. J. Fishman (New York: AIP), 365
 Covino, S., et al. 1999, *A&A*, 348, L1
 Djorgovski, S. G., Frail, D. A., Kulkarni, S. R., Bloom, J., Odewahn, S. C., & Diercks, A. 2001, *ApJL*, in press (astro-ph/0107539)
 Frail, D. A., Waxman, E., & Kulkarni, S. R. 2000, *ApJ*, 537, 191
 Freedman, D. L., & Waxman, E. 2001, *ApJ*, 547, 922
 Fruchter, A., & Metzger, M. 2001, *GCN Circ.* 1061 (<http://gcn.gsfc.nasa.gov/gcn/gcn3/1061.gcn3>)
 Fruchter, A., Vreeswijk, P., Hook, R., & Pian, E. 2000, *GCN Circ.* 752 (<http://gcn.gsfc.nasa.gov/gcn/gcn3/752.gcn3>)
 Fruchter, A. S., et al. 1999, *ApJ*, 519, L13
 Ghisellini, G., & Lazzati, D. 1999, *MNRAS*, 309, L7
 Groot, P. J., et al. 1998, *ApJ*, 502, L123
 Guetta, D., Spada, M., & Waxman, E. 2001, *ApJ*, 557, 399
 Halpern, J. P., & Fesen, R. 1998, *GCN Circ.* 134 (<http://gcn.gsfc.nasa.gov/gcn/gcn3/134.gcn3>)
 Halpern, J. P., et al. 2000, *ApJ*, 543, 697
 Harrison, F. A., et al. 1999, *ApJ*, 523, L121
 ———. 2001, *ApJ*, 559, 123
 Jaunsen, A. O., et al. 2001, *ApJ*, 546, 127
 Jensen, B. L., Hjorth, J., Pedersen, H., Kristen, H. E., Tomassi, L., Pian, E., & Hurley, K. 1999, *GCN Circ.* 454 (<http://gcn.gsfc.nasa.gov/gcn/gcn3/454.gcn3>)
 Kobayashi, S., & Sari, R. 2001, *ApJ*, 551, 934
 Kulkarni, S. R., et al. 1998, *Nature*, 393, 35
 ———. 1999, *Nature*, 398, 389
 Kumar, P. 2000, *ApJ*, 538, L125
 Kumar, P., & Panaitescu, A. 2000, *ApJ*, 541, L51
 Kumar, P., & Piran, T. 2000, *ApJ*, 535, 152
 Li, Z.-Y., & Chevalier, R. A. 2001, *ApJ*, 551, 940
 Masetti, N., et al. 2000, *A&A*, 354, 473
 Moderski, R., Sikora, M., & Bulik, T. 2000, *ApJ*, 529, 151
 Panaitescu, A., & Kumar, P. 2001, *ApJ*, 554, 667
 Phinney, E. S. 1991, *ApJ*, 380, L17
 Pian, E., et al. 2001, *A&A*, 372, 456
 Price, P. A., et al. 2001, *ApJ*, 549, L7
 Reichart, D. E., Lamb, D. Q., Fenimore, E. E., Ramirez-Ruiz, E., Cline, T. L., & Hurley, K. 2001, *ApJ*, 552, 57
 Rhoads, J. E. 1997, *ApJ*, 487, L1
 ———. 1999, *ApJ*, 525, 737
 Sari, R. 1999, *ApJ*, 524, L43
 Sari, R., Piran, T., & Halpern, J. P. 1999, *ApJ*, 519, L17
 Schmidt, M. 2001, *ApJ*, 552, 36
 Stanek, K. Z., et al. 2001, *ApJ*, in press (astro-ph/0104329)
 Stanek, K. Z., Garnavich, P. M., Kaluzny, J., Pych, W., & Thompson, I. 1999, *ApJ*, 522, L39
 Stratta, G., et al. 2000, in *Fifth Huntsville Symp. on Gamma-Ray Bursts*, ed. R. M. Kippen, R. S. Mallozzi, & G. J. Fishman (New York: AIP), 375
 Wang, X. Y., Dai, Z. G., & Lu, T. 2000, *MNRAS*, 317, 170
 Waxman, E., Kulkarni, S. R., & Frail, D. A. 1998, *ApJ*, 497, 288
 Wijers, R. A. M. J., et al. 1999, *ApJ*, 523, L33